Cost Function of Leontief Production Function

The production function of a firm is given by $f(x_1, x_2) = \min\{2\sqrt{x_1}, \sqrt{x_2}\}$. The factor prices w_1 and w_2 and the output price p are fixed.

1. Explore whether the production function exhibits increasing returns to scale.
2. Determine the cost function.
3. How much will the firm produce?

1 Solution

1. For all $t \geq 1$, we have

$$f(tx_1, tx_2) = \min\{2\sqrt{tx_1}, \sqrt{tx_2}\} = \sqrt{t}\min\{2\sqrt{x_1}, \sqrt{x_2}\} = \sqrt{t}f(x_1, x_2).$$

   The production exhibits decreasing returns to scale because the homogeneity grade is $1/2$ and $1/2 < 1$.

2. The optimal factor input satisfies

$$2\sqrt{x_1} = \sqrt{x_2} \Rightarrow 4x_1 = x_2.$$

   We get

$$y = f(x_1, 4x_1) = \min\{2\sqrt{x_1}, \sqrt{4x_1}\} = \min\{2\sqrt{x_1}, 2\sqrt{x_1}\} = 2\sqrt{x_1} \Rightarrow x_1(y) = \frac{y^2}{4}$$

   and $x_2(y) = \frac{1}{4}x_1(y) = y^2$. Hence, the cost function is given by $C(y) = w_1\frac{y^2}{4} + w_2y^2$.

3. The profit function of the firm is given by $\pi(y) = py - C(y)$. The first order condition yields

$$\pi'(y) = p - w_1\frac{y}{2} - 2w_2y = 0 \Rightarrow 2p = (w_1 + 4w_2)y \Rightarrow y = \frac{2p}{w_1 + 4w_2}.$$